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Mining Frequent Patterns without Candidate Generation [*]

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Abstract

Mining frequent patterns in transaction databases, time-series databases, and many other kinds of databases has been studied popularly in data mining research. Most of the previous studies adopt an Apriori-like candidate set generation-and-test approach. However, candidate set generation is still costly, especially when there exist long patterns.

In this study, we propose a novel frequent pattern tree (FP-tree) structure, which is an extended prefix-tree structure for storing compressed, crucial information about frequent patterns, and develop an efficient FP-tree-based mining method, FP-growth, for mining *the complete set of frequent patterns* by pattern fragment growth. Efficiency of mining is achieved with three techniques: (1) a large database is compressed into a highly condensed, much smaller data structure, which avoids costly, repeated database scans, (2) our FP-tree-based mining adopts a pattern fragment growth method to avoid the costly generation of a large number of candidate sets, and (3) a partitioning-based divide-and-conquer method is used to dramatically reduce the search space. Our performance study shows that the FP-growth method is efficient and scalable for mining both long and short frequent patterns, and is about an order of magnitude faster than the Apriori algorithm and also faster than some recently reported new frequent pattern mining methods.

1 Introduction

Frequent pattern mining [3, 14] plays an essential role in mining associations [21, 19, 23, 11, 17, 15, 8, 20, 25, 6, 18, 26], correlations [7, 22], sequential patterns [4, 24], episodes [16], multi-dimensional patterns [13], max-patterns [5], partial periodicity [10], emerging patterns [9], and many other important data mining tasks.

Most of the previous studies adopt an Apriori-like approach, whose essential idea is to iteratively generate the set of candidate patterns of length $(k+1)$ from the set of frequent patterns of length k (for $k \geq 1$), and check their corresponding occurrence frequencies in the database. An important heuristic adopted in these methods, called Apriori *heuristic* [3], which may greatly reduce the size of candidate pattern set, is the *anti-monotonicity* property of frequent sets [3, 18]: *if any length k pattern is not frequent in the database, its length $(k+1)$ super-pattern can never be frequent.*

The Apriori heuristic achieves good performance gain by (possibly significantly) reducing the size of candidate sets. However, in situations with prolific frequent patterns, long patterns, or quite low minimum support thresholds, an Apriori-like algorithm may still suffer from the following two nontrivial costs:

- It is costly to handle a huge number of candidate sets. For example, if there are 10^4 frequent 1-itemsets, the Apriori algorithm will need to generate more than 10^7 length-2 candidates and accumulate and test their occurrence frequencies. Moreover, to discover a frequent pattern of size 100, such as $\{a_1, \ldots, a_{100}\}$, it will need $\binom{100}{1}$ length-1 candidates, $\binom{100}{2}$ length-2 candidates, and so on, and the total number

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of candidates needed is

$$\binom{100}{1} + \binom{100}{2} + \cdots + \binom{100}{100} = 2^{100} - 1 \approx 10^{30}$$

This is the inherent cost of candidate generation approach, no matter what implementation technique is applied to try to optimize its detailed processing.

- It is tedious to repeatedly scan the database and check a large set of candidates by pattern matching, which is especially true for mining long patterns.

Is there any other way that one may avoid these major costs in frequent pattern mining? Can one construct some novel data structure to reduce such costs? This is the motivation of this study, especially for mining databases containing a mixture of large numbers of long and short patterns.

After some careful examinations, we believe that the bottleneck of the Apriori method is at the *candidate set generation and test.* If one can avoid generating a huge set of candidate patterns, the performance of frequent pattern mining can be substantially improved.

This problem is attacked in the following three aspects.

First, we design a novel data structure, called *frequent pattern tree*, or FP-tree for short, which is an extended prefix-tree structure storing crucial, quantitative information about frequent patterns. To ensure that the tree structure is compact and informative, only frequent length-1 items will have nodes in the tree. The tree nodes are arranged in such a way that more frequently occurring nodes will have better chances of sharing nodes than less frequently occurring ones. Our experiments show that such a tree is highly compact, usually orders of magnitude smaller than the original database. This offers an FP-tree-based mining method a much smaller data set to work on.

Second, we develop an FP-tree-based pattern fragment growth mining method, which starts from a frequent length-1 pattern (as an initial *suffix pattern*), examines only its *conditional pattern base* (a "sub-database" which consists of the set of frequent items co-occurring with the suffix pattern), constructs its (*conditional*) FP-tree, and performs mining recursively with such a tree. The pattern growth is achieved via concatenation of the suffix pattern with the new ones generated from a conditional FP-tree. Since the frequent itemset in any transaction is always encoded in the corresponding path of the frequent pattern trees, pattern growth ensures the completeness of the result. In this context, our method is not Apriori-like *restricted generation-and-test* but *restricted test only.* The major operations of mining are count accumulation and prefix path count adjustment, which are usually much less costly than candidate generation and pattern matching operations performed in most Apriori-like algorithms.

Third, the search technique employed in mining is a *partitioning-based, divide-and-conquer method* rather than Apriori-like *bottom-up generation of frequent itemsets combinations.* This dramatically reduces the size of *conditional pattern base* generated at the subsequent level of search as well as the size of its corresponding *conditional* FP-tree. Moreover, it transforms the problem of finding long frequent patterns to looking for shorter ones and then concatenating the suffix. It employs the least frequent items as suffix, which offers good selectivity. All these techniques contribute to the substantial reduction of search costs.

To compare our approach with others, we have noticed a recent study by Agarwal et al. [2] which has proposed a novel *tree structure* technique, called *lexicographic tree*, and developed a TreeProjection algorithm for mining frequent patterns. Their study reported that the TreeProjection algorithm achieves an order of magnitude performance gain of over Apriori. A comparative analysis is offered here to compare our approach with theirs. A performance study has also been conducted to compare the performance of FP-growth with Apriori and TreeProjection. Our study shows that FP-growth is at least an order of magnitude faster than Apriori, and such a margin grows even wider when the frequent patterns grow longer, and FP-growth also outperforms the TreeProjection algorithm. Our FP-tree-based mining method has also been tested in large transaction databases in industrial applications.

The remaining of the paper is organized as follows. Section 2 introduces the FP-tree structure and its construction method. Section 3 develops an FP-tree-based frequent pattern mining algorithm, FP-growth . Section 4 presents our performance study and an analytical comparison with other frequent pattern methods. Section

5 discusses the extensions, implications, and applications of the method. Section 6 summarizes our study and points out some future research issues.

2 Frequent Pattern Tree: Design and Construction

Like most traditional studies in association mining, we define the frequent pattern mining problem as follows.

Definition 1 (Frequent pattern) Let $I = \{a_1, a_2, \ldots, a_m\}$ be a **set of items**, and a **transaction database** $DB = \langle T_1, T_2, \ldots, T_n \rangle$, where T_i $(i \in [1..n])$ is a transaction which contains a set of items in I. The **support**[1] (or occurrence frequency) of a **pattern** A, which is a set of items, is the number of transactions containing A in DB. A, is a **frequent pattern** if A's support is no less than a predefined *minimum support threshold*, ξ. □

Given a transaction database DB and a minimum support threshold, ξ, the problem of *finding the complete set of frequent patterns* is called the frequent pattern mining problem.

2.1 Frequent Pattern Tree

To design a compact data structure for efficient frequent pattern mining, let's first examine a tiny example.

Example 1 Let the transaction database, DB, be (the first two columns of) Table 1 and the minimum support threshold be 3.

Transaction ID	Items Bought	(Ordered) Frequent Items
100	f, a, c, d, g, i, m, p	f, c, a, m, p
200	a, b, c, f, l, m, o	f, c, a, b, m
300	b, f, h, j, o	f, b
400	b, c, k, s, p	c, b, p
500	a, f, c, e, l, p, m, n	f, c, a, m, p

Table 1: The transaction database DB as our running example.

A compact data structure can be designed based on the following observations.

1. Since only the frequent items will play a role in the frequent pattern mining, it is necessary to perform one scan of DB to identify the set of frequent items (with *frequency count* obtained as a by-product).
2. If we store the *set* of frequent items (i.e., notice that the ordering is unimportant) of each transaction in some compact structure, it may avoid repeated scanning of DB.
3. If multiple transactions share an identical frequent item set, they can be merged into one with the number of occurrences registered as *count*. It is easy to check whether two sets are identical if all the frequent items in different transactions are sorted according to a fixed order.
4. If two transactions share a common prefix, according to some sorted order of frequent items, the shared parts can be merged using one prefix structure as long as the *count* is registered properly. If the frequent items are sorted in their *frequency descending order*, there are better chances that more prefix strings can be shared.

With these observations, one may construct a frequent pattern tree as follows.

First, a scan of DB derives a *list* of frequent items, $\langle (f:4), (c:4), (a:3), (b:3), (m:3), (p:3) \rangle$, (the number after ":" indicates the support), and with items ordered in frequency descending order. This ordering

[1]For convenience of discussion, *support* is define here as *absolute* occurrence frequency. Notice it is defined in some literature as the *relative* one, i.e., the occurrence frequency vs. the total number of transactions in DB.

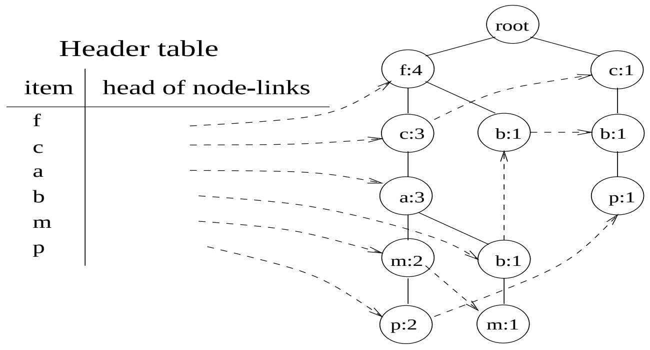


Figure 1: The FP-tree built based on the data in Table 1

is important since each path of a tree will follow this order. For convenience of later discussions, the frequent items in each transaction are listed in this ordering in the rightmost column of Table 1.

Second, one may create a root of a tree, labeled with "*null*". Scan the DB the second time. The scan of the first transaction leads to the construction of the first branch of the tree: $\langle (f:1),(c:1),(a:1),(m:1),(p:1)\rangle$. Notice that the branch is not ordered in $\langle f,a,c,m,p\rangle$ as in the transaction but is ordered according to the order in the *list* of frequent items. For the second transaction, since its (ordered) frequent item list $\langle f,c,a,b,m\rangle$ shares a common prefix $\langle f,c,a\rangle$ with the existing path $\langle f,c,a,m,p\rangle$, the count of each node along the prefix is incremented by 1, and one new node $(b:1)$ is created and linked as a child of $(a:2)$ and another new node $(m:1)$ is created and linked as the child of $(b:1)$. For the third transaction, since its frequent item list is $\langle f,b\rangle$ shares only the node $\langle f\rangle$ with the f-prefix subtree, f's count is incremented by 1, and a new node $(b:1)$ is created and linked as a child of $(f:3)$. The scan of the fourth transaction leads to the construction of the second branch of the tree, $\langle (c:1),(b:1),(p:1)\rangle$. For the last transaction, since its frequent item list $\langle f,c,a,m,p\rangle$ is identical to the first one, the path is shared with the first one, with the count of each node along the path incremented by 1.

To facilitate tree traversal, we build an item header table, in which each item points, via a head of node-link, to its first occurrence in the tree. Nodes with the same item-name are linked in sequence via such node-links. After scanning all the transactions in DB, the tree with the associated node-links is shown in Figure 1. □

This example leads to the following design and construction of a *frequent pattern tree*.

Definition 2 (FP-tree) A **frequent pattern tree** (or FP-tree in short) is a tree structure defined below.

1. It consists of one root labeled as "*null*", a set of item prefix subtrees as the children of the root, and a frequent-item header table.

2. Each node in the item prefix subtree consists of three fields: *item-name*, *count*, and *node-link*, where *item-name* registers which item this node represents, *count* registers the number of transactions represented by the portion of the path reaching this node, and *node-link* links to the next node in the FP-tree carrying the same item-name, or null if there is none.

3. Each entry in the frequent-item header table consists of two fields, (1) *item-name* and (2) *head of node-link*, which points to the first node in the FP-tree carrying the *item-name*. □

Based on this definition, we have the following FP-tree construction algorithm.

Algorithm 1 (FP-tree **construction**)

Input: A transaction database DB and a minimum support threshold ξ.

Output: Its frequent pattern tree, FP-tree

Method: The FP-tree is constructed in the following steps.

1. Scan the transaction database DB once. Collect the set of frequent items F and their supports. Sort F in support descending order as L, the *list* of frequent items.

2. Create a root of an FP-tree, T, and label it as "null". For each transaction $Trans$ in DB do the following.

   Select and sort the frequent items in $Trans$ according to the order of L. Let the sorted frequent item list in $Trans$ be $[p|P]$, where p is the first element and P is the remaining list. Call $insert\_tree([p|P], T)$.

   The function $insert\_tree([p|P], T)$ is performed as follows. If T has a child N such that $N.item\text{-}name = p.item\text{-}name$, then increment N's count by 1; else create a new node N, and let its count be 1, its parent link be linked to T, and its node-link be linked to the nodes with the same *item-name* via the node-link structure. If P is nonempty, call $insert\_tree(P, N)$ recursively.

Analysis. From the FP-tree construction process, we can see that one needs exactly two scans of the transaction database, DB: the first collects the set of frequent items, and the second constructs the FP-tree. The cost of inserting a transaction $Trans$ into the FP-tree is $O(|Trans|)$, where $|Trans|$ is the number of frequent items in $Trans$. We will show that the FP-tree contains the complete information for frequent pattern mining. $\square$

2.2 Completeness and Compactness of FP-tree

There are several important properties of FP-tree which can be observed from the FP-tree construction process.

Lemma 2.1 *Given a transaction database DB and a support threshold ξ, its corresponding* FP-tree *contains the complete information of DB in relevance to frequent pattern mining.*

Rationale. Based on the FP-tree construction process, each complete set of frequent items in a transaction T in the DB is recorded in one path of the tree, with the item occurrence information registered in the count of each corresponding node. That is, each transaction in the DB is mapped to one path in the FP-tree, and the frequent itemset information in each transaction is completely stored in the FP-tree. Moreover, one path in the FP-tree may represent frequent itemsets in multiple transactions without ambiguity since the path representing every transaction must start from the root of each item prefix subtree. Thus we have the lemma. $\square$

Based on this lemma, after an FP-tree for DB is constructed, only the FP-tree is needed in the remaining of the mining process, regardless of the number and length of the frequent patterns.

Lemma 2.2 *Without considering the (null) root, the size of an* FP-tree *is bounded by the overall occurrences of the frequent items in the database, and the height of the tree is bounded by the maximal number of frequent items in any transaction in the database.*

Rationale. Based on the FP-tree construction process, for any transaction T in DB, there exists a path in the FP-tree starting from the corresponding item prefix subtree so that the set of nodes in the path is exactly the same set of frequent items in T. Since no frequent item in any transaction can create more than one node in the tree, the root is the only extra node created not by frequent item insertion, and each node contains one node-link and one count information, we have the bound of the size of the tree stated in the Lemma. The height of any p-prefix subtree is the maximum number of frequent items in any transaction with p appearing at the head of its frequent item list. Therefore, the height of the tree is bounded by the maximal number of frequent items in any transaction in the database, if we do not consider the additional level added by the root. $\square$

Lemma 2.2 shows an important benefit of FP-tree: the size of an FP-tree is bounded by the size of its corresponding database because each transaction will contribute at most one path to the FP-tree, with the length equal to the number of frequent items in that transaction. Since there are often a lot of sharing of frequent items among transactions, the size of the tree is usually much smaller than its original database. Unlike

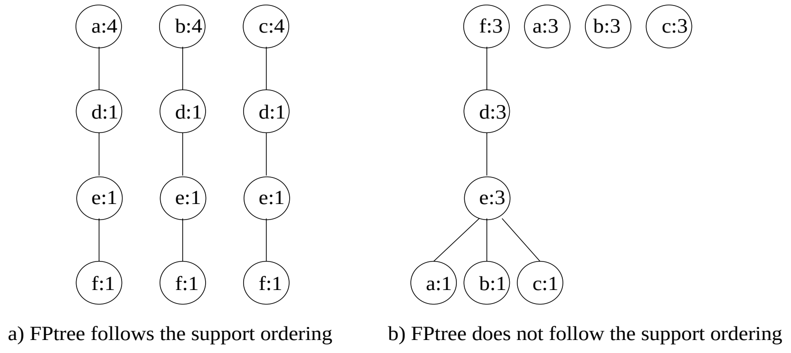

a:4
b:4
c:4
d:1
d:1
d:1
e:1
e:1
e:1
f:1
f:1
f:1
f:3
a:3
b:3
c:3
d:3
e:3
a:1
b:1
c:1
a) FPtree follows the support ordering
b) FPtree does not follow the support ordering

Figure 2: FP-tree constructed based on frequency descending ordering may not always be minimal.

the Apriori-like method which may generate an exponential number of candidates in the worst case, under no circumstances, may an FP-tree with an exponential number of nodes be generated.

FP-tree is a highly compact structure which stores the information for frequent pattern mining. Since a single path "$a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_n$" in the a_1-prefix subtree registers all the transactions whose maximal frequent set is in the form of "$a_1 \rightarrow a_2 \rightarrow \cdots \rightarrow a_k$" for any $1 \leq k \leq n$, the size of the FP-tree is substantially smaller than the size of the database and that of the candidate sets generated in the association rule mining.

Can we achieve even better compression of the original database than the FP-tree for frequent pattern mining? Let's have an analysis.

For any transaction T_i in DB, only the set of frequent items F_i will be useful for frequent pattern mining. Thus only F_i need to be recorded in the FP-tree. Since all the frequent items in T_i should be preserved as a *set* in frequent pattern mining, it is necessary to store all the items in F_i in one path in the FP-tree.

Let the set of frequent items of another transaction T_j be F_j. If $F_i = F_j$, they can be stored as one identical path, with count information registered. Thus it is necessary to register the *count* information since it saves redundant storage for patterns. If F_i and F_j share the same prefix, their common prefix should be shared, and the counts associated with the nodes along the prefix path should be accumulated.

It is essential to have the set of frequent items of each transaction starting at the root of an item prefix subtree because it avoids ambiguous interpretation of frequent patterns in different transactions. For example, a path $\langle (a_1:4) \rightarrow (a_2:3) \rightarrow (a_3:2) \rangle$ allows only one interpretation: It registers four (maximal) sets of frequent items in four transactions, which are a_1, a_1a_2, $a_1a_2a_3$, and $a_1a_2a_3$, respectively. Otherwise (i.e., if not starting at the root), there will be quite a few ambiguous interpretations, which will lead to the generation of erroneous frequent patterns.

The items in the frequent item set are ordered in the support-descending order: More frequently occurring items are arranged closer to the top of the FP-tree and thus are more likely to be shared. This indicates that FP-tree structure is usually highly compact. However, this does not mean that the tree so constructed achieves maximal compactness all the time. With the knowledge of particular data characteristics, it is possible to achieve better compression. Consider the following example. Let the transactions be: $\{adef, bdef, cdef, a, a, a, b, b, b, c, c, c\}$, and the minimum support threshold be 3. The frequent item set associated with support count becomes $\{a:4, b:4, c:4, d:3, e:3, f:3\}$. Following the item frequency ordering $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f$, the FP-tree constructed will contain 12 nodes, as shown in Figure 2 a). However, following another item ordering $f \rightarrow d \rightarrow e \rightarrow a \rightarrow b \rightarrow c$, it will contain only 9 nodes, as shown in Figure 2 b).

Our experiments also show that a small FP-trees is resulted by compressing some quite large database. For example, for the database *Connect-4* used in MaxMiner [5], which contains 67,557 transactions with 43 items in each transaction, when the support threshold is 50% (which is used in the MaxMiner experiments [5]), the total number of occurrences of frequent items is 2,219,609, whereas the total number of nodes in the FP-tree is 13,449 which represents a reduction ratio of 165.04, while it withholds hundreds of thousands of frequent patterns! (Notice that for databases with mostly short transactions, the reduction ratio is not that high.) Therefore, it is not surprising some gigabyte transaction database containing many long patterns may even generate an

FP-tree which fits in main memory.

Nevertheless, one cannot assume that an FP-tree can always fit in main memory for any large databases. An FP-tree can be partitioned and structured in the form similar to B+-tree. Such a structure will make it highly scalable to very large databases without much performance penalty. This will be discussed in Section 5.

3 Mining Frequent Patterns using FP-tree

Construction of a compact FP-tree ensures that subsequent mining can be performed in a rather compact data structure. However, this does not automatically guarantee that subsequent mining will be highly efficient since one may still encounter the combinatorial problem of candidate generation if we simply use this FP-tree to generate and check all the candidate patterns.

In this section, we will study how to explore the compact information stored in an FP-tree and develop an efficient mining method for frequent pattern mining. Although there are many kinds of frequent patterns that can be mined using FP-tree, this study will focus only on the most popularly studied one [3]: mining *all patterns*, i.e., the complete set of frequent patterns. Methods for mining other frequent patterns, such as *max-pattern* [5], i.e., those not subsumed by other frequent patterns, will be covered by subsequent studies.

We first observe some interesting properties of the FP-tree structure which will facilitate frequent pattern mining.

Property 3.1 (Node-link property) *For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the* FP-tree *header.*

This property is based directly on the FP-tree construction process. It facilitates the access of all the pattern information related to a_i by traversing the FP-tree once following a_i's node-links.

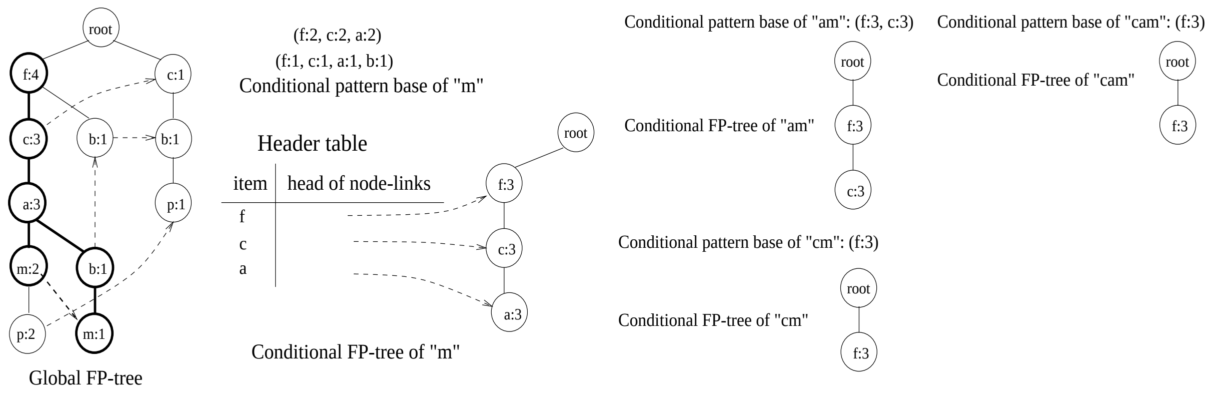
To facilitate the understanding of other FP-tree properties related to mining, we first go through an example which performs mining on the constructed FP-tree (Figure 1) in Example 1.

Example 2 Let us examine the mining process based on the constructed FP-tree shown in Figure 1. Based on Property 3.1, we collect all the patterns that a node a_i participates by starting from a_i's head (in the header table) and following a_i's node-links. We examine the mining process by starting from the bottom of the header table.

For node p, it derives a frequent pattern $(p:3)$ and two paths in the FP-tree : $\langle f:4, c:3, a:3, m:2, p:2\rangle$ and $\langle c:1, b:1, p:1\rangle$. The first path indicates that string "(f,c,a,m,p)" appears twice in the database. Notice although string $\langle f,c,a\rangle$ appears three times and $\langle f\rangle$ itself appears even four times, they only appear twice together with p. Thus to study which string appear together with p, only p's prefix path $\langle f:2, c:2, a:2, m:2\rangle$ counts. Similarly, the second path indicates string "(c,b,p)" appears once in the set of transactions in DB, or p's prefix path is $\langle c:1, b:1\rangle$. These two prefix paths of p, "$\{(f:2, c:2, a:2, m:2),\ (c:1, b:1)\}$", form p's sub-pattern base, which is called p's conditional pattern base (i.e., the sub-pattern base under the condition of p's existence). Construction of an FP-tree on this conditional pattern base (which is called p's conditional FP-tree) leads to only one branch $(c:3)$. Hence only one frequent pattern $(cp:3)$ is derived. (Notice that a pattern is an itemset and is denoted by a string here.) The search for frequent patterns associated with p terminates.

For node m, it derives a frequent pattern $(m:3)$ and two paths $\langle f:4, c:3, a:3, m:2\rangle$ and $\langle f:4, c:3, a:3, b:1, m:1\rangle$. Notice p appears together with m as well, however, there is no need to include p here in the analysis since any frequent patterns involving p has been analyzed in the previous examination of p. Similar to the above analysis, m's conditional pattern base is, $\{(f:2, c:2, a:2),\ (f:1, c:1, a:1, b:1)\}$. Constructing an FP-tree on it, we derive m's conditional FP-tree , $\langle f:3, c:3, a:3\rangle$, a single frequent pattern path. Then one can call FP-tree-based mining recursively, i.e., call $mine(\langle f:3, c:3, a:3\rangle|m)$.

From Figure 3, one can see that "$mine(\langle f:3, c:3, a:3\rangle|m)$" involves mining three items (a), (c), (f) in sequence. The first derives a frequent pattern $(am:3)$, and a call "$mine(\langle f:3, c:3\rangle|am)$"; the second derives a frequent pattern $(cm:3)$, and a call "$mine(\langle f:3\rangle|cm)$"; and the third derives only a frequent pattern $(fm:3)$. Further recursive call of "$mine(\langle f:3, c:3\rangle|am)$" derives (1) a frequent pattern $(cam:3)$, (2) a recursive call "$mine(\langle f:3\rangle|am)$" which derives a frequent pattern $(fam:3)$, and (3) another recursive call



root
f:4
c:1
c:3
b:1
b:1
a:3
p:1
m:2
b:1
p:2
m:1
Global FP-tree
(f:2, c:2, a:2)
(f:1, c:1, a:1, b:1)
Conditional pattern base of "m"
Header table
item
head of node-links
f
c
a
root
f:3
c:3
a:3
Conditional FP-tree of "m"
Conditional pattern base of "am": (f:3, c:3)
root
Conditional FP-tree of "am"
f:3
c:3
Conditional pattern base of "cm": (f:3)
root
Conditional FP-tree of "cm"
f:3
Conditional pattern base of "cam": (f:3)
root
Conditional FP-tree of "cam"
f:3

Figure 3: Example of mining process using FP-tree.

item	conditional pattern base	conditional FP-tree
p	$\{(f:2, c:2, a:2, m:2),\ (c:1, b:1)\}$	$\{(c:3)\}\|p$
m	$\{(f:4, c:3, a:3, m:2),\ (f:4, c:3, a:3, b:1, m:1)\}$	$\{(f:3, c:3, a:3)\}\|m$
b	$\{(f:4, c:3, a:3, b:1),\ (f:4, b:1),\ (c:1, b:1)\}$	$\emptyset$
a	$\{(f:3, c:3)\}$	$\{(f:3, c:3)\}\|a$
c	$\{(f:3)\}$	$\{(f:3)\}\|c$
f	$\emptyset$	$\emptyset$

Table 2: Mining of all-patterns by creating conditional (sub)-pattern bases

"$mine(\langle f:3\rangle|cam)$" which derives the longest pattern $(fcam:3)$. Similarly, the call of "$mine(\langle f:3\rangle|cm)$", derives one pattern $(fcm:3)$. Therefore, the whole set of frequent patterns involving m is $\{(m:3)$, $(am:3)$, $(cm:3)$, $(fm:3)$, $(cam:3)$, $(fam:3)$, $(fcam:3)$, $(fcm:3)\}$. This simply indicates a single path FP-tree can be mined by outputting all the combinations of the items in the path.

Similarly, node b derives $(b:3)$ and three paths: $\langle f:4, c:3, a:3, b:1\rangle$, $\langle f:4, b:1\rangle$, and $\langle c:1, b:1\rangle$. Since b's conditional pattern base: $\{(f:1, c:1, a:1)$, $(f:1)$, $(c:1)\}$ generates no frequent item, the mining terminates. Node a derives one frequent pattern $\{(a:3)\}$, and one subpattern base, $\{(f:3, c:3)\}$, a single path conditional FP-tree. Thus, its set of frequent patterns can be generated by taking their combinations. Concatenating them with $(a:3)$, we have, $\{(fa:3)$, $(ca:3)$, $(fca:3)\}$. Node c derives $(c:4)$ and one subpattern base, $\{(f:3)\}$, and the set of frequent patterns associated with $(c:3)$ is $\{(fc:3)\}$. Node f derives only $(f:4)$ but no conditional pattern base.

The conditional pattern bases and the conditional FP-trees generated are summarized in Table 2. □

The correctness and completeness of the process in Example 2 should be justified. We will present a few important properties related to the mining process of Example 2 and provide their justification.

Property 3.2 (Prefix path property) *To calculate the frequent patterns for a node a_i in a path P, only the* prefix subpath *of node a_i in P need to be accumulated, and the frequency count of every node in the prefix path should carry the same count as node a_i.*

Rationale. Let the nodes along the path P be labeled as $a_1, \ldots, a_n$ in such an order that a_1 is the root of the prefix subtree, a_n is the leaf of the subtree in P, and a_i $(1 \leq i \leq n)$ is the node being referenced. Based on the process of construction of FP-tree presented in Algorithm 1, for each prefix node a_k $(1 \leq k < i)$, the prefix subpath of the node a_i in P occurs together with a_k exactly $a_i.count$ times. Thus every such prefix node should carry the same count as node a_i. Notice that a postfix node a_m (for $i < m \leq n$) along the same path also co-occurs with node a_i. However, the patterns with a_m will be generated at the examination of the postfix node

a_m, enclosing them here will lead to redundant generation of the patterns that would have been generated for a_m. Therefore, we only need to examine the prefix subpath of a_i in P. □

For example, in Example 2, node m is involved in a path $\langle f:4, c:3, a:3, m:2, p:2\rangle$, to calculate the frequent patterns for node m in this path, only the prefix subpath of node m, which is $\langle f:4, c:3, a:3\rangle$, need to be extracted, and the frequency count of every node in the prefix path should carry the same count as node m. That is, the node counts in the prefix path should be adjusted to $\langle f:2, c:2, a:2\rangle$.

Based on this property, the prefix subpath of node a_i in a path P can be copied and transformed into a count-adjusted prefix subpath by adjusting the frequency count of every node in the prefix subpath to the same as the count of node a_i. The so transformed prefix path is called the **transformed prefixed path** of a_i for path P.

Notice that the set of transformed prefix paths of a_i form a small database of patterns which co-occur with a_i. Such a database of patterns occurring with a_i is called **a_i's conditional pattern base**, and is denoted as "*pattern_base* | a_i". Then one can compute all the frequent patterns associated with a_i in this a_i-conditional pattern_base by creating a small FP-tree, called **a_i's conditional FP-tree** and denoted as "FP-tree | a_i". Subsequent mining can be performed on this small, conditional FP-tree. The processes of construction of conditional pattern bases and conditional FP-trees have been demonstrated in Example 2.

This process is performed recursively, and the frequent patterns can be obtained by a pattern growth method, based on the following lemmas and corollary.

Lemma 3.1 (Fragment growth) *Let α be an itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then the support of $\alpha \cup \beta$ in DB is equivalent to the support of β in B.*

Rationale. According to the definition of conditional pattern base, each (sub)transaction in B occurs under the condition of the occurrence of α in the original transaction database DB. Therefore, if an itemset β appears in B ψ times, it appears with α in DB ψ times as well. Moreover, since all such items are collected in the conditional pattern base of α, $\alpha \cup \beta$ occurs exactly ψ times in DB as well. Thus we have the lemma. □

From this lemma, we can easily derive an important corollary.

Corollary 3.1 (Pattern growth) *Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is frequent in DB if and only if β is frequent in B.*

Rationale. This corollary is the case when α is a frequent itemset in DB, and when the support of β in α's conditional pattern base B is no less than ξ, the minimum support threshold. □

From the processing efficiency point of view, mining is best performed by first identifying the frequent 1-itemset, α, in DB, constructing their conditional pattern bases, and then mining the 1-itemset, β, in these conditional pattern bases, and so on. This indicates that the process of mining frequent patterns can be viewed as first mining frequent 1-itemset and then progressively growing each such itemset by mining its conditional pattern base, which can in turn be done by first mining its frequent 1-itemset and then progressively growing each such itemset by mining its conditional pattern base, etc. Thus we successfully transform a frequent k-itemset mining problem into a sequence of k frequent 1-itemset mining problems via a set of conditional pattern bases. What we need is just pattern growth. There is no need to generate any combinations of candidate sets in the entire mining process.

Finally, we provide the property on mining all the patterns when the FP-tree contains only a single path.

Lemma 3.2 (Single FP-tree path pattern generation) *Suppose an FP-tree T has a single path P. The complete set of the frequent patterns of T can be generated by the enumeration of all the combinations of the subpaths of P with the support being the minimum support of the items contained in the subpath.*

Rationale. Let the single path P of the FP-tree be $\langle a_1 : s_1 \rightarrow a_2 : s_2 \rightarrow \cdots \rightarrow a_k : s_k\rangle$. Since the FP-tree contains a single path P, the support frequency s_i of each item a_i (for $1 \leq i \leq k$) is the frequency of a_i co-occurring with its prefix string. Thus any combination of the items in the path, such as $\langle a_i, \cdots, a_j\rangle$ (for

$1 \leq i, j \leq k$), is a frequent pattern, with their co-occurrence frequency being the minimum support among those items. Since every item in each path P is unique, there is no redundant pattern to be generated with such a combinational generation. Moreover, no frequent patterns can be generated outside the FP-tree. Therefore, we have the lemma. □

Based on the above lemmas and properties, we have the following algorithm for mining frequent patterns using FP-tree and a pattern fragment growth approach.

Algorithm 2 (FP-growth : **Mining frequent patterns with** FP-tree **and by pattern fragment growth)**

Input: FP-tree constructed based on Algorithm 1, using DB and a minimum support threshold ξ.

Output: The complete set of frequent patterns.

Method: Call FP-growth (FP-tree , $null$), which is implemented as follows.

```
Procedure FP-growth ($Tree, \alpha$)
{
(1)     IF      $Tree$ contains a single path $P$
(2)     THEN FOR EACH combination (denoted as $\beta$) of the nodes in the path $P$ DO
(3)             generate pattern $\beta \cup \alpha$ with $support = minimum\ support\ of\ nodes\ in\ \beta$;
(4)     ELSE FOR EACH $a_i$ in the header of $Tree$ DO {
(5)             generate pattern $\beta = a_i \cup \alpha$ with $support = a_i.support$;
(6)             Construct $\beta$'s conditional pattern base and then $\beta$'s conditional FP-tree $Tree_\beta$;
(7)             IF      $Tree_\beta \neq \emptyset$
(8)             THEN Call FP-growth ($Tree_\beta, \beta$) }
}
```

Analysis. With the properties and lemmas in Sections 2 and 3, we show that the algorithm correctly finds the complete set of frequent itemsets in transaction database DB.

As shown in Lemma 2.1, FP-tree of DB contains the complete information of DB in relevance to frequent pattern mining under the support threshold ξ.

If an FP-tree contains a single path, according to Lemma 3.2, its generated patterns are the combinations of the nodes in the path, with the support being the minimum support of the nodes in the subpath. Thus we have lines (1) to (3) of the procedure. Otherwise, we construct conditional pattern base and mine its conditional FP-tree for each frequent itemset a_i. The correctness and completeness of prefix path transformation are shown in Property 3.2, and thus the conditional pattern bases store the complete information for frequent pattern mining. According to Lemmas 3.1 and its corollary, the patterns successively grown from the conditional FP-trees are the set of sound and complete frequent patterns. Especially, according to the fragment growth property, the support of the combined fragments takes the support of the frequent itemsets generated in the conditional pattern base. Therefore, we have lines (4) to (8) of the procedure. □

Let's now examine the efficiency of the algorithm. The FP-growth mining process scans the FP-tree of DB once and generates a small pattern-base B_{a_i} for each frequent item a_i, each consisting of the set of transformed prefix paths of a_i. Frequent pattern mining is then recursively performed on the small pattern-base B_{a_i} by constructing a conditional FP-tree for B_{a_i}. As reasoned in the analysis of Algorithm 1, an FP-tree is usually much smaller than the size of DB. Similarly, since the conditional FP-tree, "FP-tree $\mid a_i$", is constructed on the pattern-base B_{a_i}, it should be usually much smaller and never bigger than B_{a_i}. Moreover, a pattern-base B_{a_i} is usually much smaller than its original FP-tree, because it consists of the transformed prefix paths related to only one of the frequent items, a_i. Thus, each subsequent mining process works on a set of usually much smaller pattern bases and conditional FP-trees . Moreover, the mining operations consists of mainly prefix count adjustment, counting, and pattern fragment concatenation. This is much less costly than generation and test of a very large number of candidate patterns. Thus the algorithm is efficient.

From the algorithm and its reasoning, one can see that the FP-growth mining process is a divide-and-conquer process, and the scale of shrinking is usually quite dramatic. If the shrinking factor is around 20$\sim$100 for

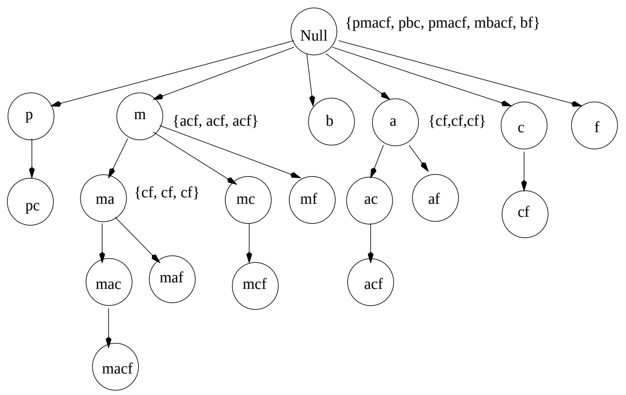


Figure 4: A lexicographical tree built for the same transactional database DB

constructing an FP-tree from a database, it is expected to be another hundreds of times reduction for constructing each conditional FP-tree from its already quite small conditional frequent pattern base.

Notice that even in the case that a database may generate an exponential number of frequent patterns, the size of the FP-tree is usually quite small and will never grow exponentially. For example, for a frequent pattern of length 100, "$a_1, \ldots, a_{100}$", the FP-tree construction results in only one path of length 100 for it, such as "$\langle a_1, \rightarrow \cdots \rightarrow a_{100} \rangle$". The FP-growth algorithm will still generate about 10^{30} frequent patterns (if time permits!!), such as "a_1, a_2, ..., a_1a_2, ..., $a_1a_2a_3$, ..., $a_1 \ldots a_{100}$". However, the FP-tree contains only one frequent pattern path of 100 nodes, and according to Lemma 3.2, there is even no need to construct any conditional FP-tree in order to find all the patterns.

4 Comparative Analysis and Performance Evaluation

In this section, we first perform a comparative analysis of FP-growth with a recently proposed lexicographical tree-based algorithm, TreeProjection [2], and then present a performance comparison of FP-growth with the classical frequent pattern mining algorithm, Apriori, and TreeProjection.

4.1 A comparative analysis of FP-growth and TreeProjection methods

The TreeProjection algorithm proposed by Agarwal et al. [2] recently is an interesting algorithm, which constructs a lexicographical tree and projects a large database into a set of reduced, item-based sub-databases based on the frequent patterns mined so far. Since it applies a tree construction method and performs mining recursively on progressively smaller databases, it shares some similarities with FP-growth. However, the two methods have some fundamental differences in tree construction and mining methodologies, and will lead to notable differences on efficiency and scalability. We will explain such similarities and differences by working through the following example.

Example 3 For the same transaction database presented in Example 1, we construct the lexicographic tree according to the method described in [2]. The result tree is shown in Figure 4, and the construction process is presented as follows.

By scanning the transaction database once, all frequent 1-itemsets are identified. As recommended in [2], the frequency ascending order is chosen as the ordering of the items. So, the order is p-m-b-a-c-f, which is exactly the reverse order of what is used in the FP-tree construction. The top level of the lexicographic tree is constructed, i.e. the root and the nodes labeled by length-1 patterns. At this stage, the root node labeled "null" and all the nodes which store frequent 1-itemsets are generated. All the transactions in the database are projected to the root node, i.e., all the infrequent items are removed.

Each node in the lexicographical tree contains two pieces of information: (i) the pattern that node represents, (ii) the set of items by adding which to the pattern may generate longer patterns. The latter piece information is recorded as *active extensions* and *active items*.

Then, a matrix at the root node is created, as shown below. The matrix computes the frequencies of length-2 patterns, thus all pairs of frequent items are included in the matrix. The items in pairs are arranged in the ordering. The matrix is built by adding counts from every transaction, i.e., computing frequent 2-itemsets based on transactions stored in the root node.

$$\begin{array}{c|cccccc} & p & m & b & a & c & f \\ p & & & & & & \\ m & 2 & & & & & \\ b & 1 & 1 & & & & \\ a & 2 & 3 & 1 & & & \\ c & 3 & 3 & 2 & 3 & & \\ f & 2 & 3 & 2 & 3 & 3 & \end{array}$$

At the same time of building the matrix, transactions in the root are projected to level-1 nodes as follows. Let $t = a_1 a_2 \cdots a_n$ be a transaction with all items listed in ordering. t is projected to node a_i $(1 \leq i < n-1)$ as $t'_{a_i} = a_{i+1} a_{i+2} \cdots a_n$.

From the matrix, all the frequent 2-itemsets are found as: $\{pc, ma, mc, mf, ac, af, cf\}$. The nodes in lexicographic tree for them are generated. At this stage, the only nodes for 1-itemsets which are active are those for m and a, because only they contain enough descendants to potentially generate longer frequent itemsets. All nodes up to and including level-1 except for these two nodes are pruned.

In the same way, the lexicographic tree is grown level by level. From the matrix at node m, nodes labeled mac, maf, and mcf are added, and only ma is active in all the nodes for frequent 2-itemsets. It is easy to see that the lexicographic tree in total contains 19 nodes. □

The number of nodes in a lexicographic tree is exactly that of the frequent itemsets. TreeProjection proposes an efficient way to enumerate frequent patterns. The efficiency of TreeProjection can be explained by two main factors: (1) the transaction projection limits the support counting in a relatively small space, and only related portions of transactions are considered; and (2) the lexicographical tree facilitates the management and counting of candidates and provides the flexibility of picking efficient strategy during the tree generation phase as well as transaction projection phase. [2] reports that their algorithm is up to one order of magnitude faster than other recent techniques in literature.

However, in comparison with the FP-growth method, TreeProjection may still suffer from some problems related to efficiency, scalability, and implementation complexity. We analyze them as follows.

First, TreeProjection may still encounter difficulties at computing matrices when the database is huge, when there are a lot of transactions containing many frequent items, and/or when the support threshold is very low. This is because in such cases there often exist a large number of frequent items. The size of the matrices at high level nodes in the lexicographical tree can be huge, as shown in our introduction section. The study in TreeProjection [2] has developed some smart memory caching methods to overcome this problem. However, it could be wise not to generate such huge matrices at all instead of finding some smart caching techniques to reduce the cost. Moreover, even if the matrix can be cached efficiently, its computation still involves some nontrivial overhead. To compute a matrix at node P with n projected transactions, the cost is $O(\sum_{i=1}^{n} \frac{|T_i|^2}{2})$, where $| T_i |$ is the length of the transaction. If the number of transaction is large and the length of each transaction is long, the computation is still costly. The FP-growth method will never need to build up matrices and compute 2-itemset frequency since it avoids the generation of any candidate k-itemsets for any k by applying a pattern growth method. Pattern growth can be viewed as successive computation of frequent 1-itemset (of the database and conditional pattern bases) and assembling them into longer patterns. Since computing frequent 1-itemsets is much less expensive than computing frequent 2-itemsets, the cost is substantially reduced.

Second, since one transaction may contain many frequent itemsets, one transaction in TreeProjection may be projected many times to many different nodes in the lexicographical tree. When there are many long transactions containing numerous frequent items, transaction projection becomes an nontrivial cost of TreeProjection . The

FP-growth method constructs FP-tree which is a highly compact form of transaction database. Thus both the size and the cost of computation of conditional pattern bases, which corresponds roughly to the compact form of projected transaction databases, are substantially reduced.

Third, TreeProjection creates one node in its lexicographical tree for each frequent itemset. At the first glance, this seems to be highly compact since FP-tree does not ensure that each frequent node will be mapped to only one node in the tree. However, each branch of the FP-tree may store many "hidden" frequent patterns because of the potential generations of many combinations using its prefix paths. Notice that the total number of frequent k-itemsets can be very large in a large database or when the database has quite long frequent itemsets. For example, for a frequent itemset $(a_1, a_2, \cdots, a_{100})$, the number of frequent itemsets at the 50th-level of the lexicographic tree will be $\begin{pmatrix} 100 \\ 50 \end{pmatrix} = \frac{100!}{50! \times 50!} \approx 1.0 \times 10^{29}$. For the same frequent itemset, FP-tree and FP-growth will only need one path of 100 nodes.

In summary, FP-growth mines frequent itemsets by (1) constructing highly compact FP-trees which share numerous "projected" transactions and hide (or carry) numerous frequent patterns, and (2) applying progressive pattern growth of frequent 1-itemsets which avoids the generation of any potential combinations of candidate itemsets implicitly or explicitly, whereas TreeProjection must generate candidate 2-itemsets for each projected database. Therefore, FP-growth is more efficient and more scalable than TreeProjection, especially when the number of frequent itemsets becomes really large. These observations and analyses are well supported by our experiments reported in this section.

4.2 Performance study

In this subsection, we report our experimental results on the performance analysis of FP-growth in comparison with Apriori and TreeProjection (on scalability and processing efficiency). It shows that FP-growth outperforms other previously proposed algorithms and is efficient and scalable in frequent pattern mining in large databases.

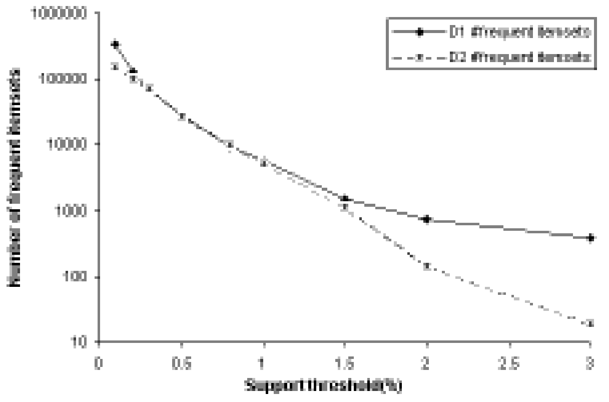
All the experiments are performed on a 450-MHz Pentium PC machine with 128 megabytes main memory, running on Microsoft Windows/NT. All the programs are written in Microsoft/Visual C++6.0. Notice that we do not directly compare our absolute number of runtime with those in some published reports running on the RISC workstations because different machine architectures may differ greatly on the absolute runtime for the same algorithms. Instead, we implement their algorithms to the best of our knowledge based on the published reports on the same machine and compare in the same running environment. Please also note that *run time* used here means the total execution time, i.e., the period between input and output, instead of *CPU time* measured in the experiments in some literature. We feel that run time is a more comprehensive measure since it takes the total running time consumed as the measure of cost, whereas CPU time considers only the cost of the CPU resource. Also, all reports on the runtime of FP-growth include the time of constructing FP-trees from the original databases.

4.2.1 Data sets

The synthetic data sets which we used for our experiments were generated using the procedure described in [3]. We refer readers to it for more details on the generation of data sets.

We report experimental results on two data sets. The first one is T25.I10.D10K with 1K items, which is denoted as $\mathcal{D}_1$. In this data set, the average transaction size and average maximal potentially frequent itemset size are set to 25 and 10, respectively, while the number of transactions in the dataset is set to 10K. The second data set, denoted as $\mathcal{D}_2$, is T25.I20.D100K with 10K items.

Some features of the two test data sets are given in Figure 5. There are exponentially numerous frequent itemsets in both data sets, as the support threshold goes down. There are pretty long frequent itemsets as well as a large number of short frequent itemsets in them. They contain abundant mixtures of short and long frequent itemsets.



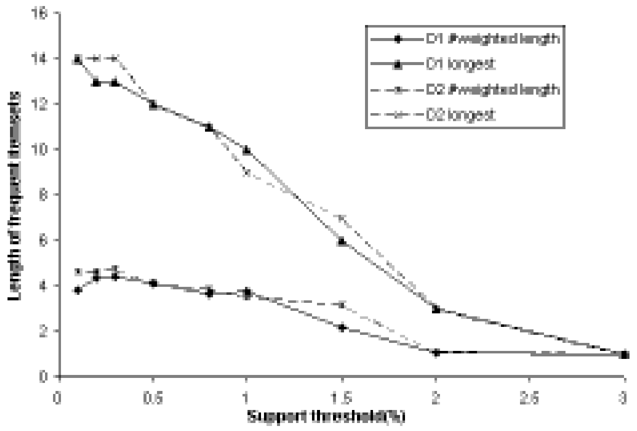
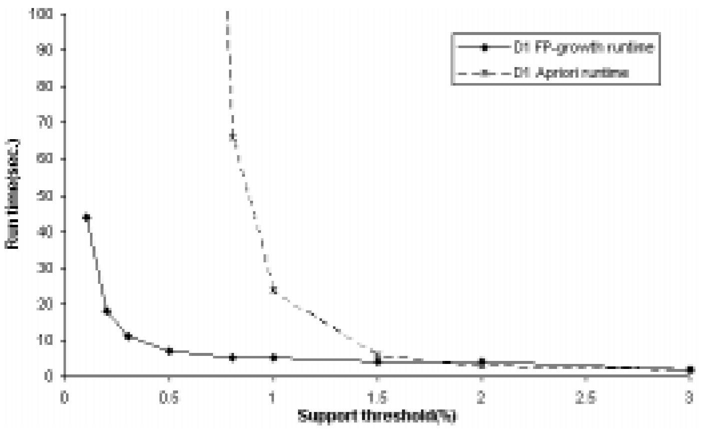


Figure 5: Features of data sets.

4.2.2 Comparison of FP-growth and Apriori

The scalability of FP-growth and Apriori as the support threshold decreases from 3% to 0.1% is shown in Figure 6.



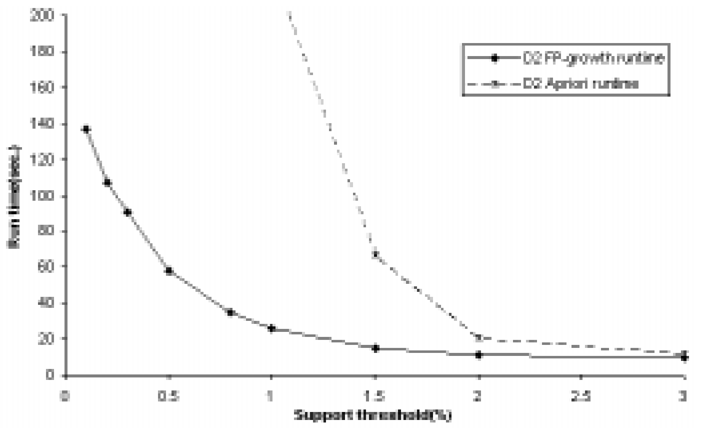


Figure 6: Scalability with threshold.

It is easy to see that FP-growth scales much better than Apriori. This is because as the support threshold goes down, the number as well as the length of frequent itemsets increase dramatically. This gives Apriori a hard time. The candidate sets that Apriori must handle becomes extremely large, and the pattern matching with a lot of candidates by searching through the transactions becomes very expensive.

Figure 7 shows that the run time per itemset of FP-growth. It shows that FP-growth has good scalability with the reduction of minimum support threshold. Although the number of frequent itemsets grows exponentially, the run time of FP-growth increases in a much more conservative way. Figure 7 indicates as the support threshold goes down, the run time per itemset decreases dramatically (notice rule time in the figure is in exponential scale). This is why the FP-growth can achieve good scalability with the support threshold.

One may concern the running memory requirements of FP-tree. As can be seen in previous sections, conditional search needs a stack of FP-trees. However, as we analyzed before, the sizes of conditional FP-trees shrink quickly. As shown in Figure 8, the running memory requirement of FP-growth is scalable.

The experimental results show that the running memory requirement of FP-growth increases linearly without exponential explosion when the support threshold goes down. This makes FP-growth a scalable algorithm for large databases. Notice that $\mathcal{D}_2$ has much more items. Thus, it has much more distinct itemsets and leads to

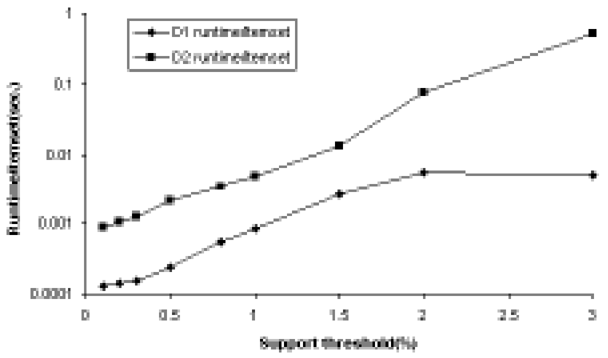


Figure 7: Run time per itemset versus support threshold.

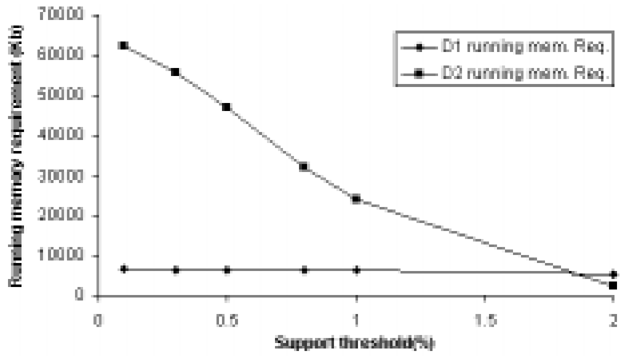


Figure 8: Running memory requirements of FP-growth .

larger FP-trees .

Please note that the running memory requirement is a mixture of *main memory* and *secondary memory*. As the conditional search deepening, the FP-trees close to the bottom of the stack can be moved to the secondary memory. Many memory management strategies can be applied to FP-growth implementation to speed up the processing.

However, for Apriori, as the size of candidate sets increases exponentially while the support threshold goes down, the running memory requirement of Apriori is exponential.

To test the scalability with the number of transactions, experiments on data set $\mathcal{D}_2$ are used. The support threshold is set to 1.5%. The results are presented in Figure 9.

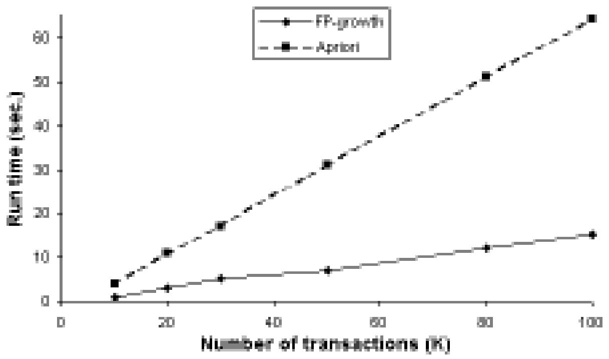


Figure 9: Scalability with number of transactions.

From the figure, one can see that both FP-growth and Apriori algorithms show linear scalability with the number of transactions from 10K to 100K. However, FP-growth is much more scalable than Apriori. As the number of transactions grows up, the difference between the two methods becomes larger and larger. Overall, FP-growth is about an order of magnitude faster than Apriori in large databases, and this gap grows wider when the minimum support threshold reduces.

4.2.3 Comparison of FP-growth and TreeProjection

As briefly introduced in Section 4.1, TreeProjection is an interesting algorithm recently proposed in [2]. We implemented a memory-based version of TreeProjection based on the techniques reported in [2]. Our implementation does not deal with *cache blocking*, which was proposed as an efficient technique when the matrix is too large to fit in main memory. However, our experiments are conducted on data sets in which all matrices as well as the lexicographic tree can be held in main memory (with our 128 mega-bytes main memory machine). We believe based on such constraints, the performance data are in general comparable and fair. Please note that the experiments reported in [2] use different datasets and different machine platforms. Thus it makes little sense to directly compare the absolute numbers reported here with [2].

According to our experimental results, both TreeProjection and FP-growth are very efficient in mining frequent patterns. Both run much faster than Apriori, especially when the support threshold is pretty low. Thus, it is inappropriate to draw all the three curves in one figure since it will make the curves of FP-growth and TreeProjection crowded together. However, a close study shows that FP-growth is better than TreeProjection when support threshold is very low and database is quite large.

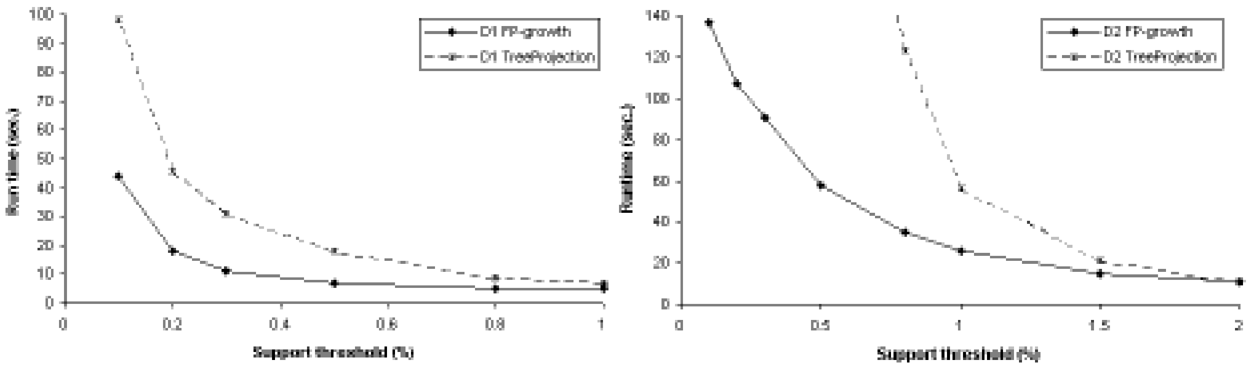


Figure 10: Scalability with support threshold.

As shown in Figure 10, both FP-growth and TreeProjection has good performance when the support threshold is pretty low, but FP-growth is better. As shown in Figure 11, in which the support threshold is set to 1%, both FP-growth and TreeProjection have linear scalability with number of transactions, but FP-growth is more scalable.

The main costs in TreeProjection are computing of matrices and transaction projections. In a database with a large number of frequent items, the matrices can become quite large, and is to compute. Also, in large databases, transaction projection may become quite costly. The height of FP-tree is limited by the length of transactions, and each branch of an FP-tree shares many transactions with the same prefix paths in the tree, which saves nontrivial costs. This explains why FP-growth has distinct advantages when the support threshold is low and when the number of transactions is large.

5 Discussions

In this section, we briefly discuss the issues on how to design a disk-resident FP-tree and how to further improve its performance. Also, we provide some additional comments on FP-tree-based mining, including materialization of FP-tree, incremental updates of FP-tree, FP-growth with item constraints, and mining other frequent patterns using this methodology.

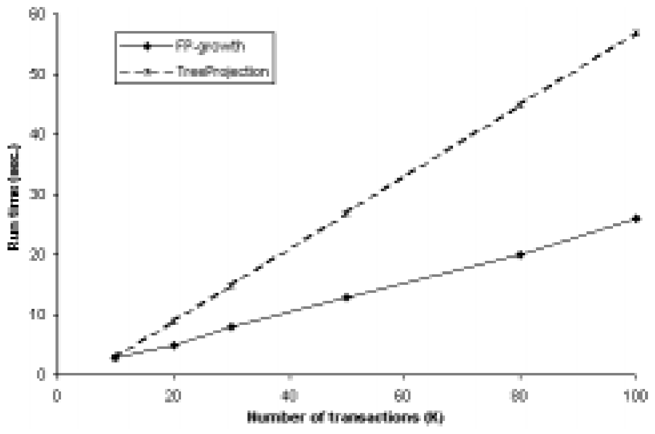


Figure 11: Scalability with number of transactions.

5.1 Disk-resident FP-tree and performance improvement

As discussed before, although in many cases, the size of an FP-tree can fit in main memory even for rather large databases, one cannot assume that an FP-tree will never grow out of the capacity of main memory. An important issue is how to structure an FP-tree to make it highly scalable.

We have the following methods to make FP-tree a disk-based structure and improve its scalability.

1. Clustering of FP-tree nodes by path and by item prefix sub-tree.

   Since there are many operations localized to single paths or individual item prefix sub-trees, such as pattern matching for node insertion, creation of transformed prefix paths for each node a_i, etc., it is important to cluster FP-tree nodes according to the tree/subtree structure. That is, (1) store each item prefix sub-tree on the same page, if possible, or at least on a sequence of continuous pages on disk; (2) store each subtree on the same page, and put the shared prefix path as the header information of the page, and (3) cluster the node-links belonging to the same paged nodes together, etc. This also facilitates a breadth-first search fashion for mining all the patterns starting from all the nodes in the header in parallel.

2. B+-tree can be constructed for FP-tree which does not fit into main memory.

   For an FP-tree with a huge number of item prefix sub-trees, a B+-tree structure can be constructed to use the roots of item prefix sub-trees to split high levels of the B+-tree, and so on. Notice when more than one page are needed to store a prefix sub-tree, the information related to the root of the subtree (or the shared prefix paths close to the top) need to be registered as header information of the page to avoid extra page access to fetch such frequently needed crucial information.

3. Mining should be performed in a group accessing mode to reduce I/O cost.

   That is, when accessing nodes following node-links, one should try to exhaust all the traversal tasks of pages in main memory before fetching and accessing other nodes in the pages on disks.

4. Space taken by a conditional pattern base or a conditional FP-tree should be released immediately after its usage.

   Since a conditional pattern base or a conditional FP-tree is associated with a particular condition which will not be used anymore when mining under such a condition is finished. Reclaiming such space immediately will reduce memory requirements and also the costs of accessing other nodes.

5. FP-tree without node-links.

   It is possible to construct an FP-tree without any node-links. In such FP-trees, one cannot follow any node links in the construction of conditional pattern-bases, but can follow each path in each item prefix subtree to project all the prefix subpaths of all the nodes into the corresponding conditional pattern bases. This is feasible if both FP-tree and its one-level conditional pattern bases can fit in memory. Otherwise, additional I/Os will be needed to swap in and out the conditional pattern bases, which is thus infeasible for large databases. Also, such a structure does not facilitate mining FP-trees with particular item constraints (see the subsection below). Thus, this design was not selected in our implementation and testing.

5.2 Additional comments on FP-tree-based mining

Before concluding our study, we would like to make some additional comments on FP-tree-based frequent pattern mining.

1. Materialization of an FP-tree. Based on our analysis, though an FP-tree is rather compact, construction of the tree needs two scans of a transaction database, which may represent a nontrivial overhead. Therefore, it could be beneficial to materialize an FP-tree for regular frequent pattern mining.

   One difficulty for FP-tree materialization is how to select a good minimum support threshold ξ in materialization since ξ is usually query-dependent. To overcome this difficulty, one may use a low ξ that may usually satisfy most of the mining queries in the FP-tree construction. For example, if we notice that 98% queries have $\xi \geq 20$, we may choose $\xi = 20$ as the FP-tree materialization threshold: that is, only 2% of queries may need to construct a new FP-tree . Since an FP-tree is organized in the way that less frequently occurring items are located at the deeper paths of the tree, it is easy to select only the upper portions of the FP-tree (or drop the low portions which do not satisfy the support threshold) when mining the queries with higher thresholds. Actually, one can directly work on the materialized FP-tree by starting at an appropriate header entry since one just need to get the (same) prefix paths no matter how low support the original FP-tree is.

2. Incremental updates of an FP-tree. The second difficulty related to FP-tree materialization is how to incrementally update an FP-tree, such as when adding daily new transactions into a database containing accumulated transaction records for months.

   If the minimum support of the materialized FP-tree is 1 (i.e., it is just a compact version of the original database in the context of frequent pattern mining), the update will not cause any problem since adding new records is equivalent to scanning additional transactions in the FP-tree construction. However, this will often result in an undesirably large FP-tree . Otherwise, if the frequency of every single items are registered in F_1 and tracked in updates (whose cost should be pretty reasonable), an FP-tree can be used to its maximal extent as follows.

   Suppose an FP-tree was constructed based on a validity support threshold (called "watermark") $\psi = 0.1\%$ in a DB with 10^8 transactions. Suppose an additional 10^6 transactions are added in. The frequency of each item is updated. If the highest relative frequency among the items not in FP-tree goes up to, say 12%, the watermark will need to go up accordingly to $\psi > 0.12\%$ to exclude such item(s). However, with more transactions added in, the watermark may even drop since an item's relative support frequency may drop with more transactions added in. Only when the FP-tree watermark is raised to some undesirable level, the reconstruction of the FP-tree for the new DB becomes necessary.

3. FP-tree mining with item constraints. Pushing constraint with Apriori-like methodology has been studied popularly [25, 18]. With FP-tree, it is easy to perform pattern growth mining by incorporating some constraints associated with a set of items. For example, instead of mining frequent patterns for all the frequent items, one may just like to mine frequent patterns only related to a particular set of items, S, such as *mining the set of frequent patterns containing c or m in Example 1.* With the same FP-tree, the FP-growth mining method may just need to be modified minorly. The only additional care is when computing a transformed prefix path for an item m, one also needs to look down the path to include the items, such as p, which are not in S. Our previous computation for the whole database will not need to consider m's pairing with p since it would have been checked when examining node p. However, since p is not in S now, such a pair would have been missed if m's computation did not look down the path to include p. FP-tree mining with sophisticated constraints, such as those in [18], will be discussed in subsequent studies.

4. FP-tree mining of other frequent patterns. FP-tree-based mining method can be extended to mining many other kinds of frequent patterns, such as mining partial periodicity in time series database, max-patterns, multi-level patterns, sequential patterns, etc. Our preliminary implementation and study [12] tell us that FP-tree-based, fragment growth mining can be applied to mining partial periodicity in time series database and to mining max-patterns efficiently. It is an interesting future research issue on how to use it at mining a large variety of interesting frequent patterns.

6 Conclusions

We have proposed a novel data structure, *frequent pattern tree* (FP-tree), for storing compressed, crucial information about frequent patterns, and developed a pattern growth method, FP-growth, for efficient mining of frequent patterns in large databases.

There are several advantages of FP-growth over other approaches: (1) It constructs a highly compact FP-tree, which is usually substantially smaller than the original database, and thus saves the costly database scans in the subsequent mining processes. (2) It applies a pattern growth method which avoids costly candidate sets generation and test by successively concatenating frequent 1-itemset found in the (conditional) FP-trees : It never generates any combinations of new candidate sets which are not in the database because the itemset in any transaction is always encoded in the corresponding path of the FP-trees . In this context, the mining methodology is not Apriori-like (*restricted*) *generation-and-test* but *frequent pattern* (*fragment*) *growth only*. The major operations of mining are count accumulation and prefix path count adjustment, which are usually much less costly than candidate generation and pattern matching operations performed in most Apriori-like algorithms. (3) It applies a partitioning-based divide-and-conquer method which dramatically reduces the size of the subsequent conditional pattern bases and conditional FP-trees. Several other optimization techniques, including ordering of frequent items, and employing the least frequent events as suffix, also contribute to the efficiency of the method.

We have implemented the FP-growth mining method, studied its performance in comparison with several influential frequent pattern mining algorithms in large databases. Our study shows that with the FP-tree structure, the mining of long and short frequent patterns, can be performed efficiently, with better performance than the existing candidate pattern generation-based algorithms. The FP-growth method has also been implemented in the new version of DBMiner system and been tested in large industrial databases, such as in London Drugs databases, with satisfactory performance

There are a lot of interesting research issues related to FP-tree-based mining, including further study and implementation of SQL-based, highly scalable FP-tree structure, constraint-based mining of frequent patterns using FP-trees, and the extension of the FP-tree-based mining method for mining other interesting frequent patterns.

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