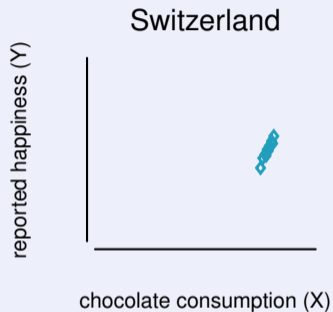
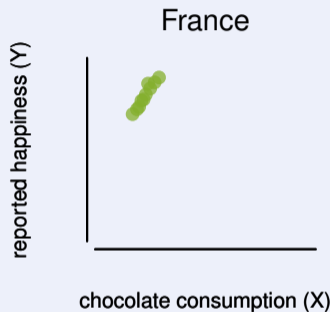


Cocoa Comfort across Contexts

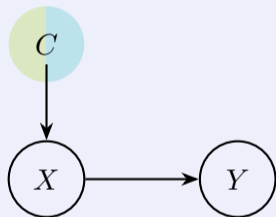


So far we assumed observations from a fixed context. But what if we obtain data from **heterogeneous data sources**?

Heterogeneous Data Sources

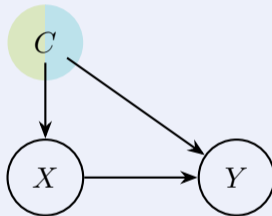
Roadmap: Tour du Chocolat

Invariance



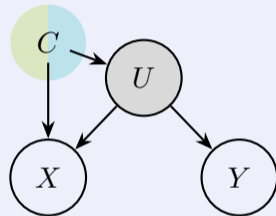
New worlds,
same rules

Independent Change



dependent samples,
independent mechanisms

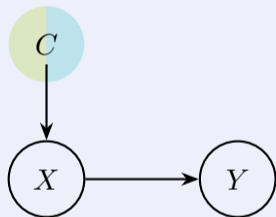
Contextual Confounding



dependent mechanisms?
... confounding!

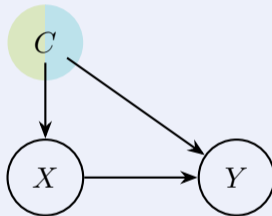
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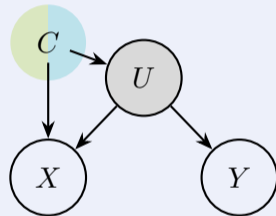
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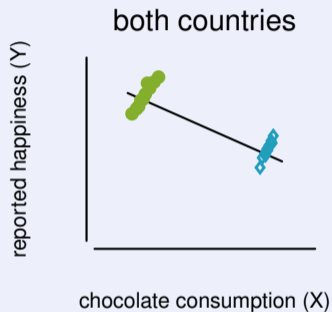
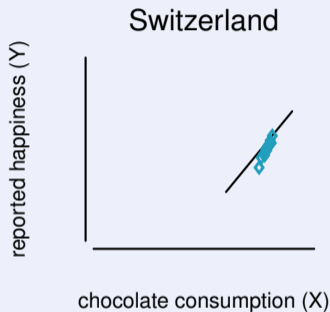
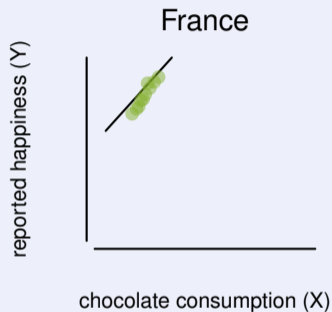
Contextual Confounding



dependent mechanisms?
... confounding!

Causal Invariance

Sweet Swiss Paradox



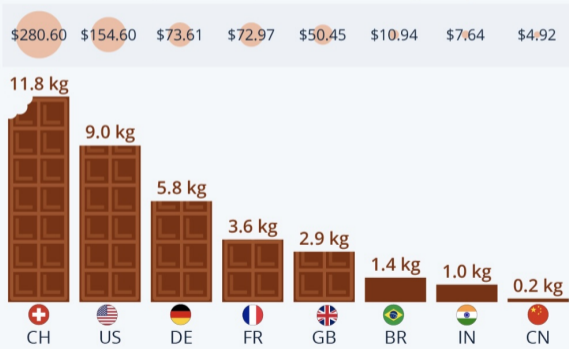
- ▶ **France, Switzerland:** More chocolate → improved happiness.
- ▶ **Both countries:** More chocolate → less happiness (?!)

Sweet Swiss Paradox, Explained

(Not) Everybody Loves Chocolate

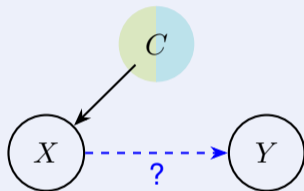
Estimated chocolate consumption per capita in selected countries in 2022

■ Per-capita consumption ● Revenue per capita



Multiple Contexts: A Puppeteer in Plain Sight

- ▶ Measured context C (countries, studies, or subpopulations).
- ▶ Here affects cause X (variation of chocolate per capita).
- ▶ Pooling data naively: averaging apples (●) and oranges (●).



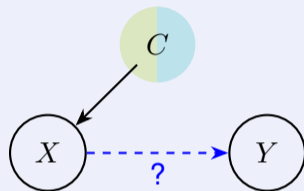
Causal Invariance

- ▶ Pooling contexts c : causal links may appear, vanish, or even reverse!
- ▶ But in each context c :

$$Y^{(c)} = \beta X^{(c)} + \varepsilon$$

- ▶ **Causal Invariance:**

$$\beta_{France} \approx \beta_{Switzerland}$$

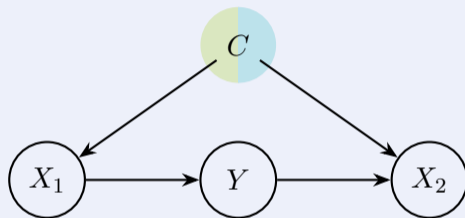


What Does Invariance Mean?

We want to find $\Phi(X)$ such that:

- ▶ $Y \perp\!\!\!\perp C \mid \Phi(X)$, and
- ▶ $\Phi(X)$ is informative about Y
- ▶ This is an optimization problem

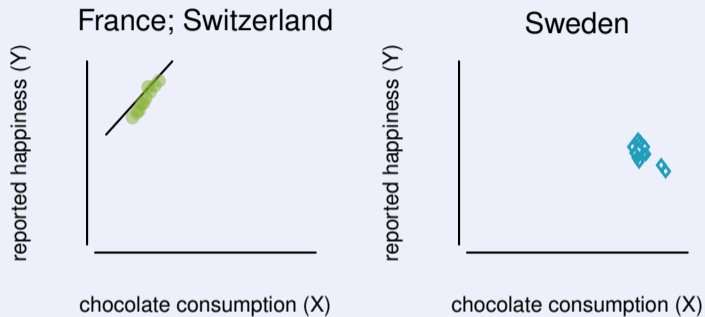
$$\min_{\Phi} (I(\Phi(X); Y) - \alpha I(C; Y \mid \Phi(X)))$$



Invariance-Based Methods

- ▶ **Causal predictors are invariant**
Causal Inference Using Invariant Prediction[Ⓞ]
- ▶ **Invariance as conditional independence of C**
CD-NOD: Causal Discovery from Heterogeneous/Nonstationary Data[Ⓞ]
- ▶ **Invariance = optimality across environments**
Invariant Risk Minimization[Ⓞ]
- ▶ **Generalization = Invariance**
Invariant Models for Causal Transfer Learning[Ⓞ]

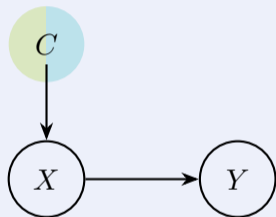
When Invariance Breaks ...



- ▶ So far, we assumed the effect of chocolate on happiness is stable.
- ▶ But what if $P(Y | X)$ changes across environments?

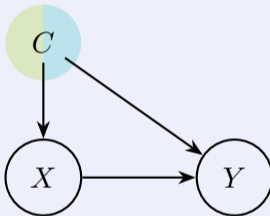
Three Layers of Distribution Shifts

Invariance



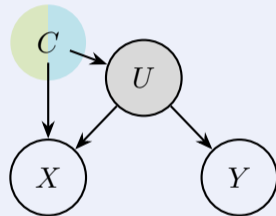
New worlds,
same rules

Independent Change



dependent samples,
independent mechanisms

Contextual Confounding

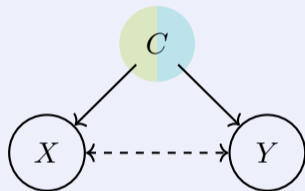


dependent mechanisms?
... confounding!

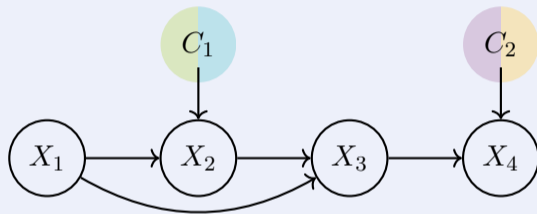
Independent Change

Beyond Invariance

- ▶ Context C .
- ▶ Affecting both predictors X and target Y .
- ▶ **Modularity**: causal mechanisms **change independently** of one another.

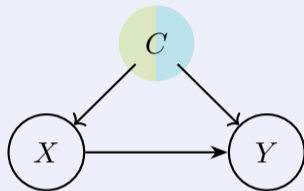


Independent Change: Nature's API Design



Independent Shifts = Sparse Shifts

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|-------|---|------|---|-------|---|------|---|
| X | Green | | | | Blue | | | |
| $Y X$ | Green | | Blue | | Green | | Blue | |
| Y | Green | | Blue | | Red | | Pink | |
| $X Y$ | Green | | Red | | Blue | | Pink | |



The true causal graph entails fewer distinct mechanisms!

Independent Mechanism Shift Methods

- ▶ **Fewer Mechanism Shifts are Better**
Causal Discovery in Heterogeneous Environments Under the Sparse Mechanism Shift Hypothesis[Ⓞ]
- ▶ **Compressing Multiple Mechanisms to Obtain Causal Graphs**
Learning Causal Models under Independent Changes[Ⓞ]
- ▶ **Exogenous Variables Can Enforce Invariance**
Anchor Regression: Heterogeneous Data Meet Causality[Ⓞ]
- ▶ **Neural Networks Can Leverage Independent Mechanism Shifts**
Learning Robust Models Using The Principle of Independent Causal Mechanisms[Ⓞ]

Recall: Not all that Glitters is a Gold Standard

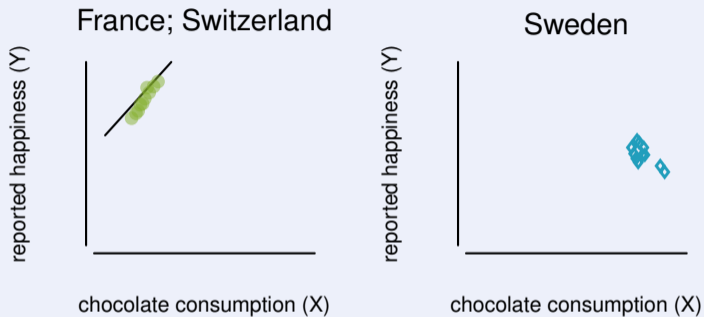
*“Despite my efforts to keep the study groups blinded, some of the participants changed groups mid-study. Indeed, those in the control group (who received no extra chocolate) decided to **start raiding** the chocolate of those in the interventional arms (dark chocolate or milk chocolate). Furthermore, participants in the dark and milk chocolate groups **traded chocolate** based on their individual preferences.” (emphasis added)*

Chen 2007, A clinical trial gone awry: the Chocolate Happiness Undergoing More Pleasantness (CHUMP) study[🔗]

The RCT We Didn't Run

- ▶ In an **ideal world**, we randomize chocolate consumption (... forcing people to eat more chocolate? Keeping people from raiding chocolate?)
- ▶ In a **nonideal world**, we observe different countries with different habits. (... some countries naturally consume more chocolate than others!)
- ▶ We didn't run an RCT — but the world kind of did.

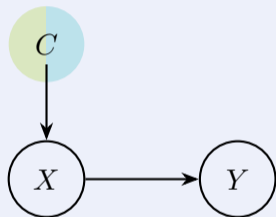
What else can we do with Nature's RCT?



- ▶ What if the changes of $P(X)$, $P(Y | X)$ across environments are dependent?
- ▶ **Enter: Hidden Confounders.**

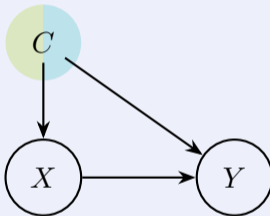
Three Layers of Distribution Shifts

Invariance



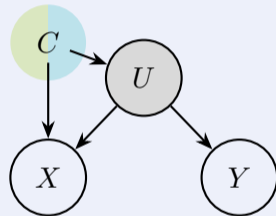
New worlds,
same rules

Independent Change



dependent samples,
independent mechanisms

Contextual Confounding



dependent mechanisms?
... confounding!

Contextual Confounders

Confounders in Disguise

- ▶ The context didn't just shift chocolate consumption — it also shifted something else:

$$\text{Wealth} \rightarrow \begin{cases} \text{more chocolate} \\ \text{higher happiness} \end{cases}$$

- ▶ Now changes in $P(X)$ and $P(Y | X)$ are **no longer independent**.

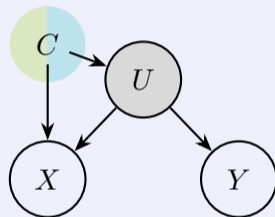
Confounders Revealed: Violations of ICM

- ▶ If both $P(X)$ and $P(Y | X)$ shift across environments...
- ▶ ...and they shift in **coordinated** ways...
- ▶ ... something is linking them behind the scenes.
- ▶ **Confounding implies correlated shifts.**

$$I_C \left(P^{(C)}(X), P^{(C)}(Y | X) \right) \neq 0 \Rightarrow \text{suspect a hidden confounder.}$$

Unmeasured Confounding Causal Correlated Mechanism Shifts

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------|-------|---|------|---|-------|---|------|---|
| U | Green | | | | Blue | | | |
| $X U$ | Green | | Blue | | Green | | Blue | |
| $Y U$ | Green | | | | | | | |
| X | Green | | Blue | | Red | | Pink | |
| Y | Green | | | | Blue | | | |



Context Confounding Compatible Approaches

- ▶ **Leveraging Invariance in Linear Models with Latent Confounders**
Causal Dantzig: fast inference in linear structural equation models with hidden variables under additive interventions[Ⓞ]
- ▶ **Running FCI on Data Including Environment Variables**
Joint Causal Inference from Multiple Contexts[Ⓞ]
- ▶ **Detecting Mechanism Shift Correlations for Confounder Detection**
Identifying Confounding from Causal Mechanism Shifts[Ⓞ]