

Question of Interest



What about **time**?

Granger "causality"

A variable X **Granger-causes** another variable Y if X values help **predicting** the future of Y

$$\mathbb{P}[Y(t+1) \in A | \mathcal{I}(t)] \neq \mathbb{P}[Y(t+1) \in A | \mathcal{I}_{-X}(t)]$$

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Not true causality

(as claimed by Granger himself)

Association level on causality ladder

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Association level on causality ladder

Limitations:

- No instantaneous effects
- Mostly for linear relationships (VAR models)

Temporal causal model

Functional causal model (FCM)

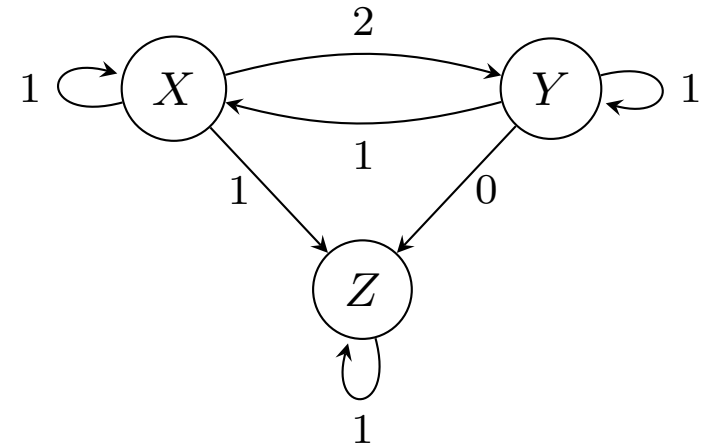
$$X_t := f_X(X_{t-1}, Y_{t-1}, \varepsilon_X)$$

$$Y_t := f_Y(Y_{t-1}, X_{t-2}, \varepsilon_Y)$$

$$Z_t := f_Z(Z_{t-1}, X_{t-1}, Y_t, \varepsilon_Z)$$

with noise terms ε jointly independent

(summary) Temporal causal graph



Joint probability distribution

$$P(X, Y, Z)$$

Temporal causal model

Functional causal model (FCM)

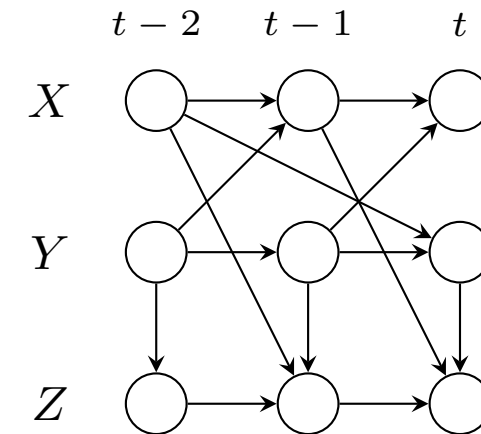
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(window) Temporal causal graph



Joint probability distribution

$$P(X, Y, Z)$$

Time series setup challenge

Non-i.i.d. data

(independence testing, likelihood computation...)

Delay identification

Non-stationarity

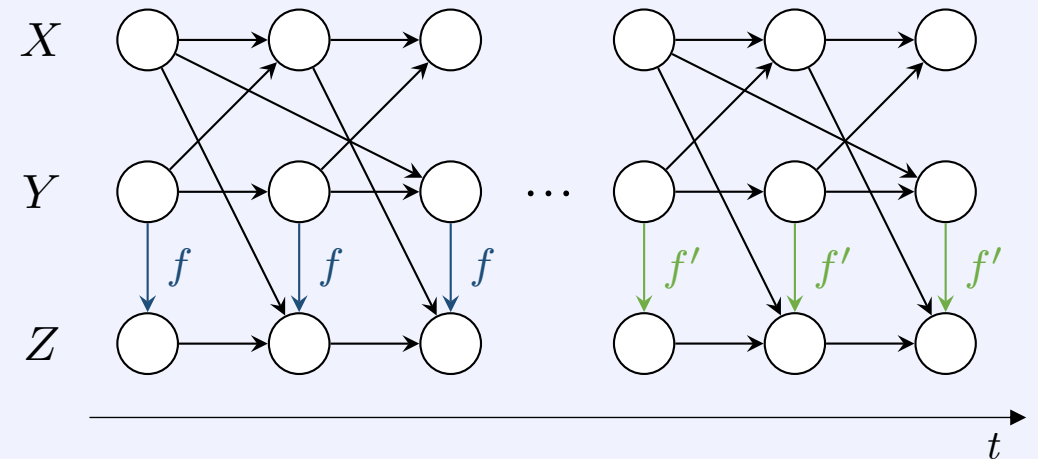
Non-stationarity under causal perspective

Origin

- Functional mechanism change
- Varying cause-effect delay
- Cause-effect pairs deletion/addition/flip

How

- Drift vs. abrupt change
- Recurrent regimes



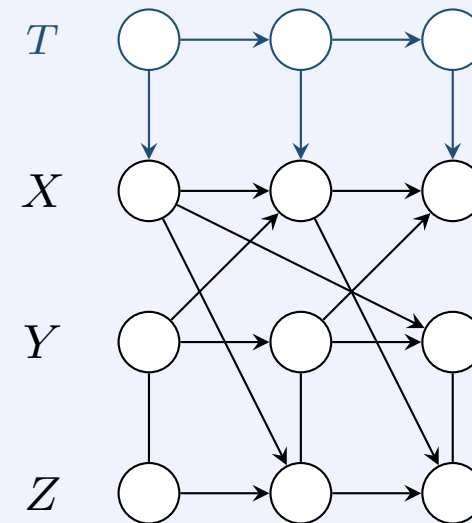
Constraint-based causal discovery from non-stationary TS

CD-NOD

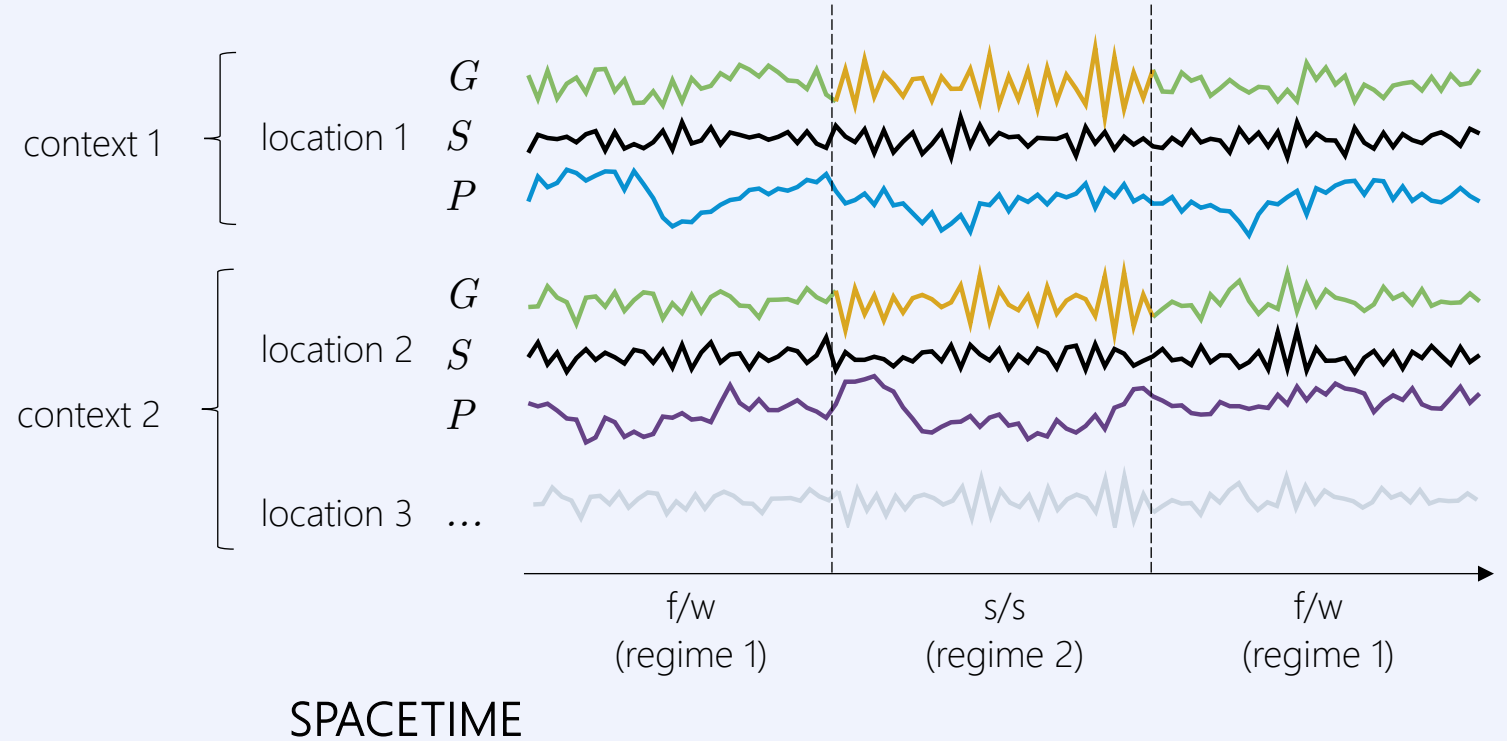
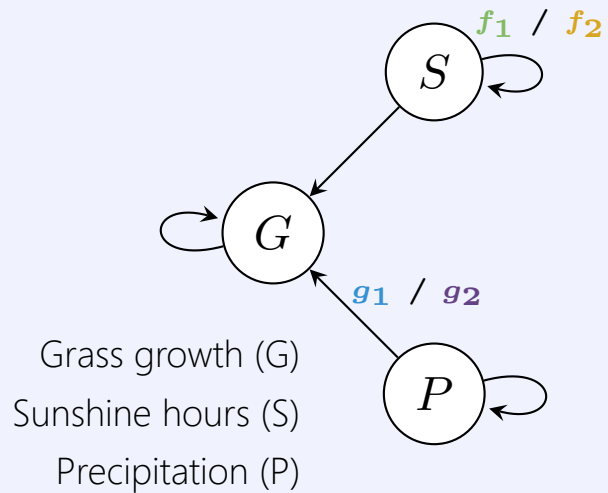
- Based on **conditional independence tests**
- Use of a **surrogate variable** to model time (time index)
- Can handle non-stationarity if it can be expressed as smooth function of time

Limitations

- Partially oriented graph
- In practice, use conditional independence test designed for i.i.d. samples and multiple statistical testing problem

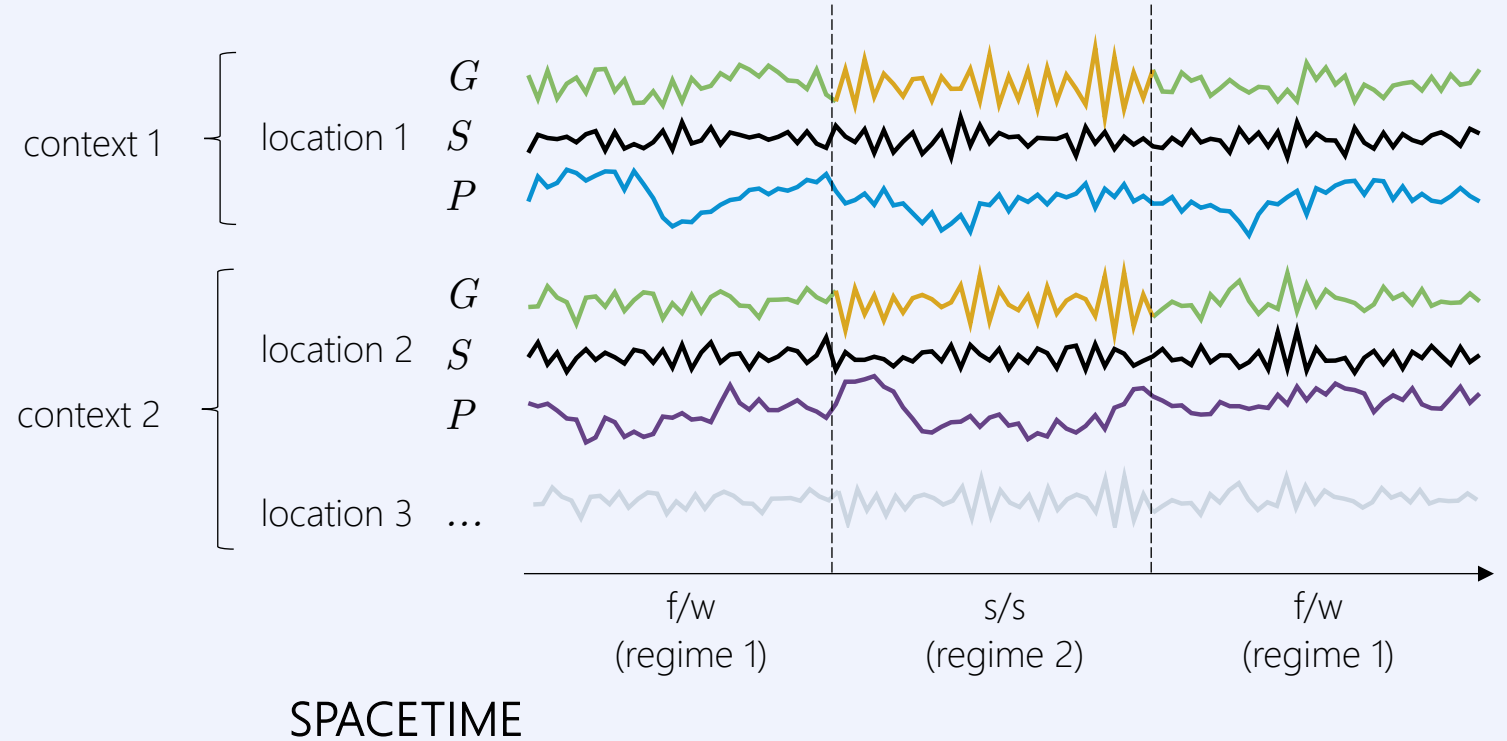
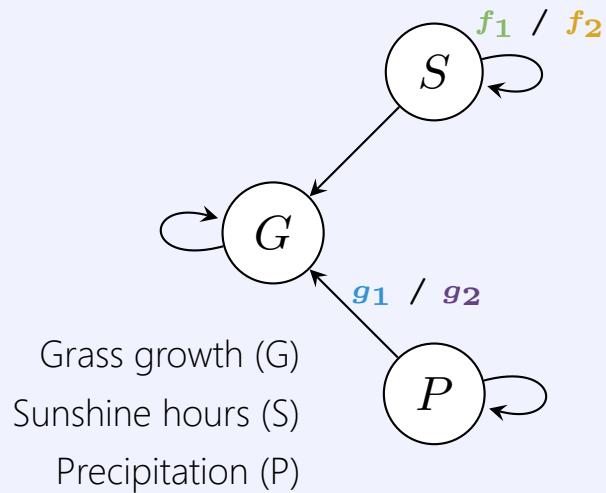


AIT-based causal discovery from non-stationary TS



Times series with **recurrent regimes** and from **heterogeneous contexts**

AIT-based causal discovery from non-stationary TS

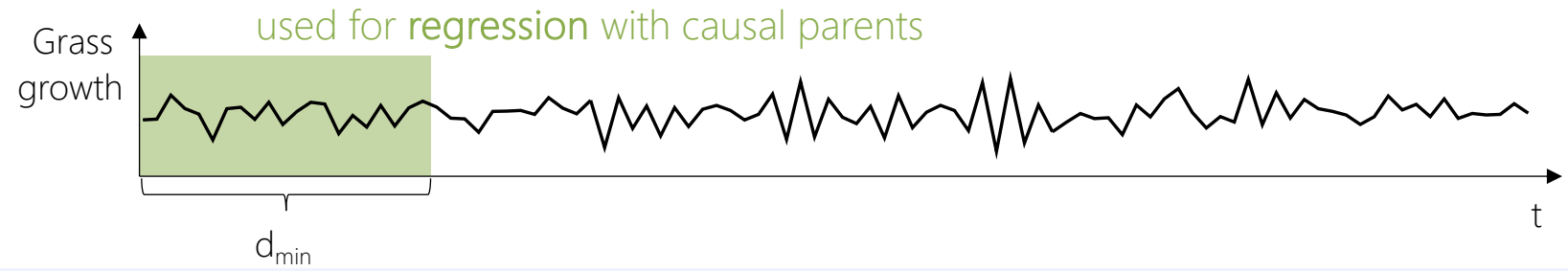


Times series with **recurrent regimes** and from **heterogeneous contexts**

Major assumption: the causal structure does not change

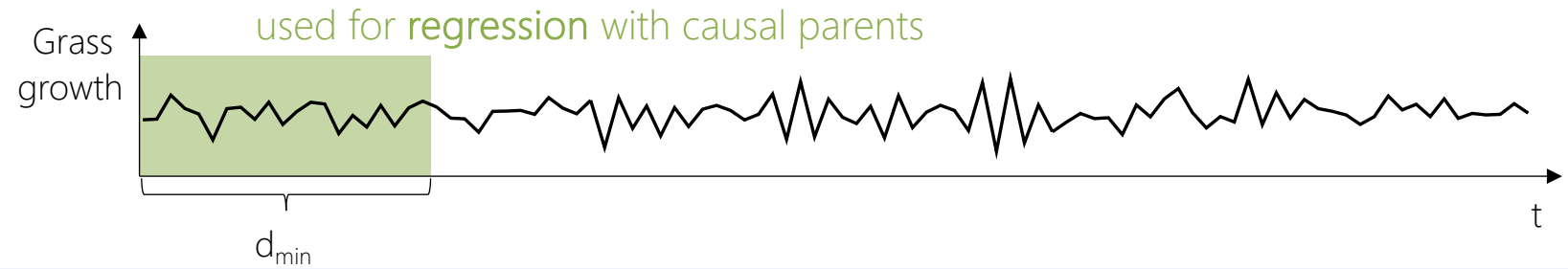
Detecting regime changes

1 Functional modeling

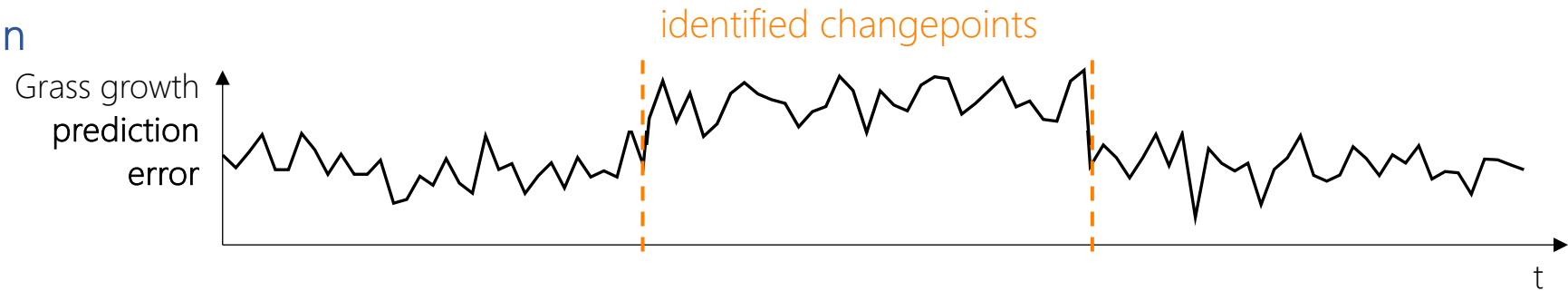


Detecting regime changes

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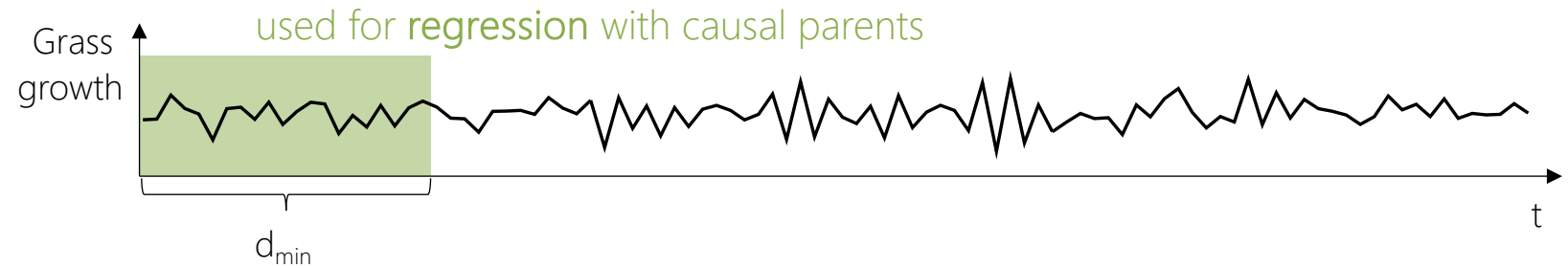


2 Changepoint detection

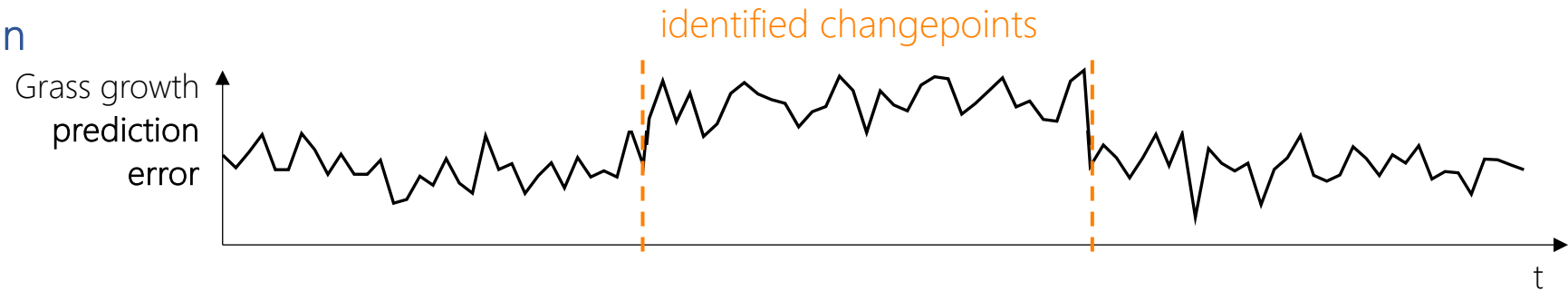


Detecting regime changes

1 Functional modeling



2 Changepoint detection



3 Regime partitioning

